RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4762

Mechanics 2

Tuesday **7 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

1 **(a)** Roger of mass 70 kg and Sheuli of mass 50 kg are skating on a horizontal plane containing the standard unit vectors **i** and **j**. The resistances to the motion of the skaters are negligible. The two skaters are locked in a close embrace and accelerate from rest until they reach a velocity of $2\mathbf{i}\ \mathrm{m\,s^{-1}}$, as shown in Fig. 1.1.

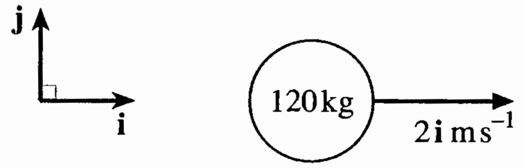


Fig. 1.1

(i) What impulse has acted on them? [1]

During a dance routine, the skaters separate on three occasions from their close embrace when travelling at a constant velocity of $2\mathbf{i}\ \mathrm{m\,s^{-1}}$.

(ii) Calculate the velocity of Sheuli after the separation in the following cases.

(*A*) Roger has velocity $\mathbf{i}\ \mathrm{m\,s^{-1}}$ after the separation.

(*B*) Roger and Sheuli have equal speeds in opposite senses after the separation, with Roger moving in the **i** direction.

(*C*) Roger has velocity $4(\mathbf{i}+\mathbf{j})\ \mathrm{m\,s^{-1}}$ after the separation. [6]

(b) Two discs with masses 2 kg and 3 kg collide directly in a horizontal plane. Their velocities just before the collision are shown in Fig. 1.2. The coefficient of restitution in the collision is 0.5.

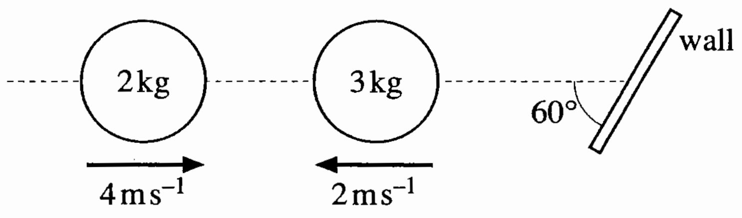


Fig. 1.2

(i) Calculate the velocity of each disc after the collision. [6]

The disc of mass 3 kg moves freely after the collision and makes a perfectly elastic collision with a smooth wall inclined at 60° to its direction of motion, as shown in Fig. 1.2.

(ii) State with reasons the speed of the disc and the angle between its direction of motion and the wall after the collision. [4]

2 A car of mass 850 kg is travelling along a road that is straight but not level.

On one section of the road the car travels at constant speed and gains a vertical height of 60 m in 20 seconds. Non-gravitational resistances to its motion (e.g. air resistance) are negligible.

(i) Show that the average power produced by the car is about 25 kW. [2]

On a *horizontal* section of the road, the car develops a constant power of exactly 25 kW and there is a constant resistance of 800 N to its motion.

(ii) Calculate the maximum possible steady speed of the car. [3]

(iii) Find the driving force and acceleration of the car when its speed is $10\,\mathrm{m\,s^{-1}}$. [3]

When travelling along the horizontal section of road, the car accelerates from $15\,\mathrm{m\,s^{-1}}$ to $20\,\mathrm{m\,s^{-1}}$ in 6.90 seconds with the same constant power and constant resistance.

(iv) By considering work and energy, find how far the car travels while it is accelerating. [6]

When the car is travelling at $20\,\mathrm{m\,s^{-1}}$ up a constant slope inclined at arcsin (0.05) to the horizontal, the driving force is removed. Subsequently, the resistance to the motion of the car remains constant at 800 N.

(v) What is the speed of the car when it has travelled a further 105 m up the slope? [5]

3 Fig. 3.1 shows an object made up as follows. ABCD is a uniform lamina of mass 16 kg. BE, EF, FG, HI, IJ and JD are each uniform rods of mass 2 kg. ABCD, BEFG and HIJD are squares lying in the same plane. The dimensions in metres are shown in the figure.

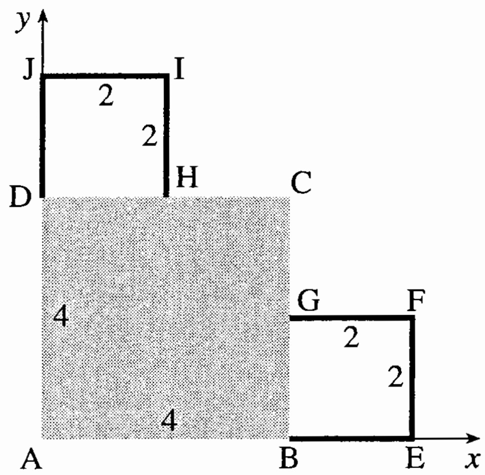


Fig. 3.1

(i) Find the coordinates of the centre of mass of the object, referred to the axes shown in Fig. 3.1. [5]

The rods are now re-positioned so that BEFG and HIJD are perpendicular to the lamina, as shown in Fig. 3.2.

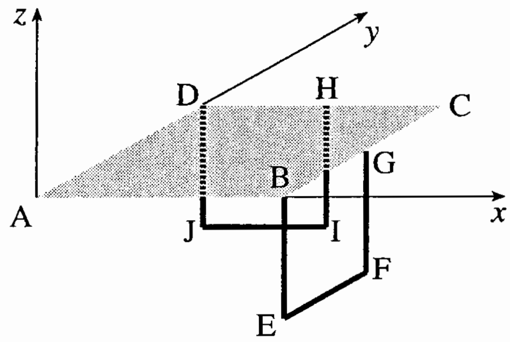


Fig. 3.2

(ii) Find the x-, y- and z-coordinates of the centre of mass of the object, referred to the axes shown in Fig. 3.2. Calculate the distance of the centre of mass from A. [8]

The object is now freely suspended from A and hangs in equilibrium with AC at $\alpha°$ to the vertical.

(iii) Calculate α. [4]

4 **(a)** A framework is made from light rods AB, BC and CA. They are freely hinged to each other at A, B and C and to a vertical wall at A. The hinge at B rests on a smooth, horizontal support. The rod AC is horizontal. A vertical load of LN acts at C. This information is shown in Fig. 4.1 together with the dimensions of the framework and the external forces UN, VN and RN acting on the framework.

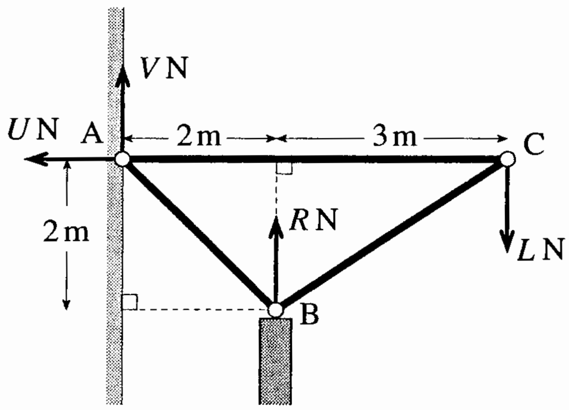


Fig. 4.1

(i) Show that $R = 2.5L$, $U = 0$ and $V = -1.5L$. [4]

(ii) Calculate the internal forces in the rods AB, AC and BC in terms of L, stating whether each of these rods is in tension or thrust (compression). [8]

(b) Fig. 4.2 shows a plank of weight W resting at the points A and B on two fixed supports. The plank is at an angle θ to the horizontal. Its centre of mass, G, is such that AG is 2 m and GB is 1 m.

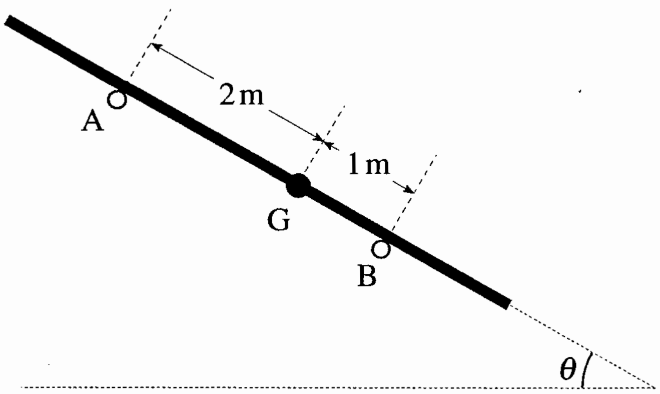


Fig. 4.2

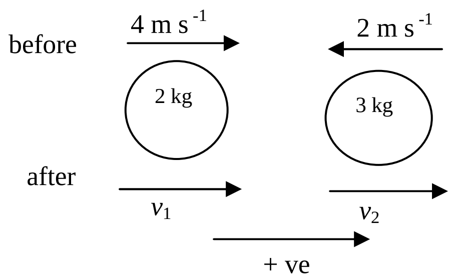
The contact between the plank and the support at A is rough, but that at B is smooth. The plank is on the point of slipping.

(i) Draw a diagram showing all the forces acting on the plank. [1]

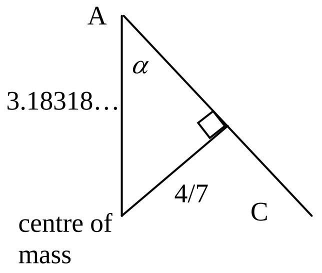
(ii) By taking moments about a suitable point, find an expression in terms of W and θ for the normal reaction at A of the support on the plank. [3]

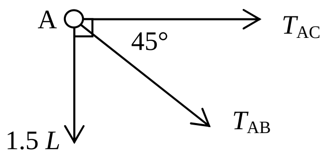
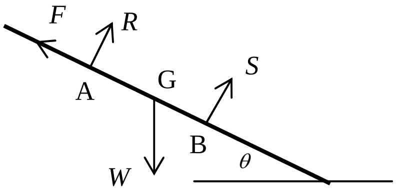
(iii) Find an expression in terms of θ for the coefficient of friction between the plank and the rough support. [3]

Mark Scheme 4762
June 2005

Q 1		mark		Sub
(a) (i)	240 **i** N s $\rightarrow$	B1		1
(ii)				
(A)	240 **i** = 70 **i** +50 **v** so **v** = 3.4 **i** m s $^{-1}$	M1 A1	Equating to **their** 240 **i** in this part FT 240 **i**	
(B)	240 **i** = 70u **i** – 50u **i**	M1	Must have u in both RHS terms and opposite signs	
	u = 12 so **v** = – 12 **i** m s $^{-1}$	A1	FT 240 **i**	
(C)	240 **i** = 280(**i** + **j**) +50$\mathbf{v}_B$	M1	FT 240 **i** Must have all terms present	
	so $\mathbf{v}_B$ = (-0.8 **i** – 5.6 **j**) m s $^{-1}$	A1	cao	6
(b) (i)				
	NEL $\dfrac{v_2 - v_1}{-2-4} = -0.5$	M1	NEL	
	so $v_2 - v_1 = 3$	A1	Any form	
	PCLM $8-6 = 2v_1 + 3v_2$	M1 A1	PCLM Any form	
	Solving v_2 = 1.6 so 1.6 m s $^{-1}$ $\rightarrow$ v_1 = – 1.4 so 1.4 m s $^{-1}$ $\leftarrow$	A1 A1	Direction must be clear (accept diagram) Direction must be clear (accept diagram). [Award A1 A0 if v_1 & v_2 correct but directions not clear]	6
(ii)	1.6 m s $^{-1}$ at 60° to the wall (glancing angles both 60°)	B1 B1	FT **their** 1.6	
	No change in the velocity component parallel to the wall as no impulse No change in the velocity component perpendicular to the wall as perfectly elastic	E1 E1	Must give reason Must give reason	4
	total	17		

Q 2		mark		Sub
(i)	We need $\frac{mgh}{t} = \frac{850 \times 9.8 \times 60}{20}$ = 24 990 so approx 25 kW	M1 E1	Use of $\frac{mgh}{t}$ Shown	2
(ii)	Driving force – resistance = 0 $25000 = 800v$ so v = 31.25 and speed is 31.25 m s $^{-1}$	B1 M1 A1	May be implied Use of $P = Fv$	3
(iii)	Force is $\frac{25000}{10} = 2500$ N N2L in direction of motion $2500 - 800 = 850a$ a = 2 so 2 m s $^{-2}$	B1 M1 A1	 Use of N2L with all terms	3
(iv)	$0.5 \times 850 \times 20^2 = 0.5 \times 850 \times 15^2$ $+25000 \times 6.90$ $-800x$ x = 122.6562… so 123 m (3 s. f.)	M1 B1 B1 B1 A1 A1	W-E equation with KE and power term One KE term correct Use of Pt .Accept wrong sign WD against resistance. Accept wrong sign All correct cao	6
(v)	**either** $0.5 \times 850 \times v^2 = 0.5 \times 850 \times 20^2$ $-850 \times 9.8 \times \frac{105}{20}$ -800×105 $v^2 = 99.452...$ so 9.97 m s $^{-1}$ **or** N2L + ve up plane $-(800 + 850g \times 0.05) = 850a$ a = -1.43117… $v^2 = 20^2 + 2 \times (-1.43117...) \times 105$ $v^2 = 99.452...$ so 9.97 m s $^{-1}$	 M1 M1 A1 B1 A1 M1 A1 M1 A1 A1	 W-E equation inc KE, GPE and WD GPE term with attempt at resolution Correct. Accept expression. Condone wrong sign. WD term. Neglect sign. cao N2L. All terms present. Allow sign errors. Accept ± Appropriate *uvast*. Neglect signs. All correct including consistent signs. Need not follow sign of a above. cao	5
		19		

Q 3		mark		Sub
(i)	$28\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 16\begin{pmatrix}2\\2\end{pmatrix} + 2\begin{pmatrix}5\\0\end{pmatrix} + 2\begin{pmatrix}6\\1\end{pmatrix} + 2\begin{pmatrix}5\\2\end{pmatrix}$ $+ 2\begin{pmatrix}0\\5\end{pmatrix} + 2\begin{pmatrix}1\\6\end{pmatrix} + 2\begin{pmatrix}2\\5\end{pmatrix}$ $\bar{x} = 2.5$ $\bar{y} = 2.5$	M1 B1 B1 A1 A1	Complete method Total mass correct 3 c. m. correct (or 4 x- or y-values correct) [Allow A0 A1 if only error is in total mass] [If $\bar{x} = \bar{y}$ claimed by symmetry and only one component worked replace final A1, A1 by B1 explicit claim of symmetry A1 for the 2.5]	5
(ii)	$\bar{x} = \bar{y}$ $28\bar{x} = 16 \times 2 + 6 \times 4 + 2 \times 0 + 2 \times 1 + 2 \times 2$ $\bar{x} = \frac{31}{14}$ (2.21428…) $\bar{z} = \frac{8 \times (-1) + 4 \times (-2)}{28} = -\frac{4}{7}$ (- 0.57142…) Distance is $\sqrt{\left(\frac{31}{14}\right)^2 + \left(\frac{31}{14}\right)^2 + \left(\frac{4}{7}\right)^2}$ = 3.18318.. so 3.18 m (3 s. f.)	B1 M1 A1 A1 A1 A1 M1 F1	Or by direct calculation Dealing with ‘folded’ parts for $\bar{x}$ or for $\bar{z}$ At least 3 terms correct for $\bar{x}$ All terms correct allowing sign errors Use of Pythagoras in 3D on **their** c.m.	8
(iii)	 $\sin\alpha = \frac{4}{7} / 3.18318..$ so α = 10.3415… so 10.3° (3 s. f.)	M1 B1 M1 A1	c.m. clearly directly below A Diagram showing α and known lengths (or equivalent). FT their values. Award if final answer follows **their** values. Appropriate expression for α. FT **their** values. cao	4
	total	17		

Q 4		mark		Sub
(a) (i)	Moments c.w. about A $2R = 5L$ so $R = 2.5L$ Resolve → $U = 0$ Resolve ↑ $V + R = L$ so $V = -1.5L$	 E1 E1 M1 E1	 Resolve vertically or take moments about B (or C)	4
(ii)	 For equilibrium at A ↑ $T_{AB}\cos 45 + 1.5L = 0$ so $T_{AB} = -\frac{3\sqrt{2}L}{2}$ so $\frac{3\sqrt{2}L}{2}$ N (C) in AB → $T_{AC} + T_{AB}\cos 45 = 0$ so $T_{AC} = \frac{3L}{2}$ so $\frac{3L}{2}$ N (T) in AC At C ↓ $L + T_{BC}\cos\theta = 0$ $\tan\theta = 3/2 \Rightarrow \cos\theta = 2/\sqrt{13}$ so $T_{BC} = -\frac{\sqrt{13}L}{2}$ so $\frac{\sqrt{13}L}{2}$ N (C) in BC	M1 M1 A1 F1 M1 B1 A1 F1	Equilibrium at a pin-joint Attempt at equilibrium at A or C including resolution with correct angle (2.12L (3 s. f.)) (1.5L) Must include attempt at angle (1.80 L (3 s. f.)) Award for T/C correct from their internal forces. Do not award without calcs	8
(b) (i)		B1	All forces present with arrows and labels. Angles and distances not required.	1
(ii)	c.w.moments about B $R \times 3 - W \times 1\cos\theta = 0$ so $R = \frac{1}{3}W\cos\theta$	 M1 A1 A1	 If moments about other than B, then need to resolve perp to plank as well Correct	3
(iii)	Resolve parallel to plank $F = W\sin\theta$ $\mu = \frac{F}{R} = \frac{W\sin\theta}{\frac{1}{3}W\cos\theta} = 3\tan\theta$	 B1 M1 A1	 Use of $F = \mu R$ and their F and R Accept any form.	3
	total	19		

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General Comments

This paper appeared to be accessible to all of the candidates, with the majority able to obtain at least some credit on some part of each question. A large number of excellent scripts were seen. There were some candidates that did not seem to appreciate that a diagram assists in finding a solution and can help to clarify the solution to the examiner. The main difficulties encountered related to giving reasons for a calculated answer or in establishing given answers. There was, from some candidates, a lack of rigour with relevant steps in working being omitted and/ or insufficient explanation as to the principles being employed. A small number of candidates penalised themselves by premature rounding of answers leading to inaccuracies in final answers.

Comments on Individual Questions

1) **Impulse and Momentum**

(a) Problems arose in this part for those candidates who did not appreciate the vector nature of the question and hence, did not specify the direction of the velocities requested.

(i) This part was almost always successfully answered.

(ii) (A) This part posed few problems for the vast majority.

(B) Many candidates obtained the correct speed for Sheuli but did not specify direction. Others set up a correct equation for Roger's speed and obtained the answer 12**i** but then failed to convert to $\mathbf{v}_s = -12\mathbf{i}$.

(C) This part was more successful with many obtaining a complete solution in terms of **i** and **j.** There were errors with signs in a number of cases.

(b)(i) Unfortunately many candidates did not draw a diagram for this part of the question and hence, errors with signs and inconsistent equations were quite frequent. Candidates could help themselves by stating which principle is being applied and specifying the meaning of the variables being employed.

(ii) This part of the question was poorly attempted by almost all of the candidates. While many of them could state that the speed would be unchanged and that the angle of reflection would be the same as the angle of incidence, few could give clear and unambiguous reasons as to why this was so. Most merely stated that the collision was perfectly elastic without expanding on what this would affect. Very few candidates seemed to appreciate the need to investigate directions parallel and perpendicular to the wall and of those that did, only a small number mentioned that there would be no impulse in the direction parallel to the wall and hence no change in that component of the velocity.

2) **Work and Energy**

Candidates either scored well on this question or very poorly.

(i) This part gave few problems to the majority of candidates although a small number of them failed to give any indication of the principles being employed and merely wrote down a set of numbers that produced the required answer.

(ii) Most candidates could gain full credit for this part.

(iii) The majority of candidates gained some credit for this part. Errors that occurred were usually due to the omission of the resistance term in the Newton's second law equation.

(iv) A sizeable number of candidates ignored the method requested in the question and attempted a solution using Newton's second law and the constant acceleration equations, obviously not appreciating that if both the power and the resistance are constant, the acceleration cannot be. Of those who used the requested method, most obtained some credit but many omitted the term involving power.

(v) Candidates who used work-energy methods for this part were on the whole more successful than those who opted for Newton's second law and *uvast*. Errors were usually the omission of one of the terms in the work-energy equation or in the sign of the acceleration in *uvast*.

3) **Centres of Mass**

This question was well done by the majority of candidates; many of them scoring highly on it. Almost all of the candidates understood the method required for finding a centre of mass and some excellent answers were seen.

(i) A high proportion of candidates could obtain the correct answer to this part of the question. However, a small number of candidates treated the shape as if it was composed of three parts, a lamina and two squares formed by rods.

(ii) A large number of candidates scored highly on this part of the question. The main errors were in the sign of the z component of the centre of mass. The majority understood that the use of Pythagoras in 3D was required to find the distance of the centre of mass from A. However, a small minority of candidates omitted this part altogether.

(iii) This part of the question gave problems to many of the candidates with only the more able candidates achieving well. Unfortunately very few candidates drew a diagram that was helpful to them. Those that drew a diagram were usually more successful in identifying the lengths necessary to calculate the requested angle and could gain some credit for their work. A significant minority did not seem to appreciate that the centre of mass of the shape had to lie directly below A.

4 **Moments and Resolution**

Some excellent responses to this question were seen but the quality of the diagrams in many cases was poor.

(a)(i) Those candidates who resolved horizontally and vertically and then took moments about A (or C) or vice versa were usually successful in showing the given results. However, a number of candidates chose to take moments about B without first establishing that $U = 0$ and omitted the moment of U.

(ii) It was pleasing to see a large number of correct responses to this part of the question. Almost all of the candidates appreciated the need to resolve at a pin-joint. Those candidates who drew a diagram showing all of the internal and external forces with clear labels were generally more successful than those who either did not draw a diagram or who drew a poor and inadequately labelled one. Without a diagram, sign errors and inconsistent equations were common. Some candidates confused tension and thrust.

(b)(i) Many poor diagrams were seen here with forces omitted or unlabelled in many of them. The most frequently omitted force was the frictional force at A and a significant minority of candidates thought that the normal reaction forces at A and B would be the same. It was common to see the weight represented as Wg.

ii) This part of the question gave little difficulty to the majority of candidates with almost all of them appreciating the need to take moments. A very small number apparently did not understand the meaning of 'normal reaction' and attempted some complex algebra to find a reaction that acted vertically upwards.

(iii) Many candidates gained significant credit on this part of the question. However, some very creative working was seen from the few who were determined to find that $\mu = \tan\theta$. This included some of the candidates that had obtained full credit for the previous part.